SIFT: Detector and Descriptor
SIFT: David Lowe, UBC
Distinctive Image Features from Scale-Invariant Keypoints

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Abstract. This paper presents a method for extracting distinctive invariant features from images that can be used to perform reliable matching between different views of an object or scene. The features are invariant to image scale and rotation, and are shown to provide robust matching across a substantial range of affine distortion, change in 3D viewpoint, addition of noise, and change in illumination. The features are highly distinctive, in the sense that a single feature can be correctly matched with high probability against a large database of features from many images. This paper also describes an approach to using these features for object recognition. The recognition proceeds by matching individual features to a database of features from known objects using a fast nearest-neighbor algorithm, followed by a Hough transform to identify clusters belonging to a single object, and finally performing verification through least-squares solution for consistent pose parameters. This approach to recognition can robustly identify objects among clutter and occlusion while achieving near real-time performance.

Keywords: invariant features, object recognition, scale invariance, image matching

1. Introduction

Image matching is a fundamental aspect of many problems in computer vision, including object or scene recognition, solving for 3D structure from multiple images, stereo correspondence, and motion tracking. This paper describes image features that have many properties that make them suitable for matching differing images of an object or scene. The features are invariant to image scaling and rotation, and partially invariant to change in illumination and 3D camera viewpoint. They are well localized in both the spatial and frequency domains, reducing the probability of disruption by occlusion, clutter, or noise. Large numbers of features can be extracted from typical images with efficient algorithms. In addition, the features are highly distinctive, which allows a single feature to be correctly matched with high probability against a large database of features, providing a basis for object and scene recognition.

The cost of extracting these features is minimized by taking a cascade filtering approach, in which the more expensive operations are applied only at locations that pass an initial test. Following are the major stages of computation used to generate the set of image features:

1. Scale-space extrema detection: The first stage of computation searches over all scales and image locations. It is implemented efficiently by using a difference-of-Gaussian function to identify potential interest points that are invariant to scale and orientation.
2. Keypoint localization: At each candidate location, a detailed model is fit to determine location and scale. Keypoints are selected based on measures of their stability.
3. Orientation assignment: One or more orientations are assigned to each keypoint location based on local image gradient directions. All future operations are performed on image data that has been transformed relative to the assigned orientation, scale, and location for each feature, thereby providing invariance to these transformations.
SIFT - Key Point Extraction

- Stands for **Scale Invariant Feature Transform**
- Patented by university of British Columbia
- Similar to the one used in primate visual system (human, ape, monkey, etc.)
- Transforms image data into scale-invariant coordinates

Goal

- Extract distinctive invariant features
  - Correctly matched against a large database of features from many images
- Invariance to image scale and rotation
- Robustness to
  - Affine (rotation, scale, shear) distortion,
  - Change in 3D viewpoint,
  - Addition of noise,
  - Change in illumination.
Advantages

- **Locality**: features are local, so robust to occlusion and clutter
- **Distinctiveness**: individual features can be matched to a large database of objects
- **Quantity**: many features can be generated for even small objects
- **Efficiency**: close to real-time performance
Invariant Local Features
Steps for Extracting Key Points

- Scale space peak selection
  - Potential locations for finding features
- Key point localization
  - Accurately locating the feature key points
- Orientation Assignment
  - Assigning orientation to the key points
- Key point descriptor
  - Describing the key point as a high dimensional vector (128) (SIFT Descriptor)
Scales

- What should be the sigma value for Canny and LoG edge detection?

- If use multiple sigma values (scales), how do you combine multiple edge maps?

- Marr-Hildreth:
  - *Spatial Coincidence* assumption:
    - Zero-crossings that coincide over several scales are physically significant.
Andrew Witkin
(July 22, 1952 – September 12, 2010)
Andrew Witkin

- The paper "Snakes: Active Contour Models" (by Kass, Witkin, and Terzopoulos) achieved an honorable mention for the Marr Prize in 1987.
  - According to CiteSeer, this paper is the 11th most cited paper ever in computer science.
- The 1987 paper "Constraints on deformable models: Recovering 3D shape and nonrigid motion" (by Terzopoulos, Witkin, and Kass) was also a prize winner.
- In 1992, Witkin and Kass where awarded the Prix Ars Electronica computer graphics award for "Reaction–Diffusion Texture Buttons."
Andrew Witkin

- Witkin received the ACM SIGGRAPH Computer Graphics Achievement Award in 2001 "for his pioneering work in bringing a physics based approach to computer graphics."

- As senior scientist at Pixar Animation Studios, Witkin received a technical academy award in 2006 for "pioneering work in physically based computer-generated techniques used to simulate realistic cloth in motion pictures."

- Andrew "Andy" Witkin died in a scuba diving accident off the coast of Monterey, California on September 12th 2010.

- In the PIXAR movie Cars 2 in the credits, exit a message: In loving memory Japhet Pieper 1965-2010 Andy Witkin 1952-2010. This message can be see at minutes:1:43:09.
SCALE-SPACE FILTERING:
A New Approach To Multi-Scale Description

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ABSTRACT

The extrema in a signal and its first few derivatives provide a useful general purpose qualitative description for many kinds of signals. A fundamental problem in competing such descriptions is scale: a derivative must be taken over some neighborhood, but there is seldom a principled basis for choosing its size. Scale-space filtering is a method that describes signals qualitatively, managing the ambiguity of scale in an organized and natural way. The signal is first expanded by convolution with Gaussian masks over a continuum of sizes. This "scale-space" image is then collapsed, using its qualitative structure, into a tree providing a concise but complete qualitative description covering all scales of observation. The description is further refined by applying a stability criterion, to identify events that persist in large changes in scale.

1. Introduction

Hardly any sophisticated signal understanding task can be performed using the raw numerical signal values directly; some description of the signal must first be obtained. An initial description ought to be as compact as possible, and its elements should correspond closely as possible to meaningful objects or events in the signal-forming process. Frequently, local extrema in the signal and its first and second derivatives— and intervals bounded by extrema—are particularly appropriate descriptive primitives: although local and closely tied to the signal data, these events often have direct semantic interpretations, e.g., as edges in images. A description that characterizes a signal by its extrema and those of its first few derivatives is a qualitative description of exactly the kind we were taught to use in elementary calculus to "sketch" a function.

A great deal of effort has been expended to obtain this kind of primitive qualitative description (for overview of this literature, see [11,12,13,14] ), and the problem has proved extremely difficult. The problem of scale has emerged consistently as a fundamental source of difficulty, because the events we perceive and find meaningful vary enormously in size and extent. The problem is not so much to eliminate low-scale noise, as to separate events at different scales arising from distinct physical processes. [7] It is possible to introduce a parameter of scale by smoothing the signal with a mask of variable size, but with the introduction of scale-dependence comes ambiguity: every setting of the scale parameter yields a different description; new extremal points may appear, and existing ones may move or disappear. How can we decide which if any of this continuum of descriptions is "right"?

There is rarely a sound basis for setting the scale parameter. In fact, it has become apparent that for many tasks no one scale of description is categorically correct: the physical processes that generate signals such as images act at a variety of scales, some intrinsically more interesting or important than another. Thus the ambiguity introduced by scale is inherent and irremovable, so the goal of scale-dependent description cannot be to eliminate this ambiguity, but rather to manage it effectively, and reduce it where possible.

This line of thinking has led to considerable interest in multi-scale descriptions [12,13,15,16]. However, merely computing descriptions at multiple scales does not solve the problem; if anything, it exacerbates it by increasing the volume of data. Some means must be found to organize or simplify the description, by relating one scale to another. Some work has been done in this area aimed at obtaining "edge pyramids" (e.g., [16]), but no clear-cut criteria for constructing them have been put forward. Marr [7] suggested that zero-crossings that coincide over several scales are "physically significant," but this idea was neither justified nor tested.

How, then, can descriptions at different scales be related to each other in an organized, natural, and compact way? Our solution, which we call scale-space filtering, begins by continuously varying the scale parameter, sweeping out a surface that we call the scale-space image. In this representation, it is possible to track extrema as they move continuously with scale changes, and to identify the singularities...
Scale Space (Witkin, IJCAI 1983)

- Apply whole spectrum of scales (sigma in Gaussian)
- Plot zero-crossings vs scales in a scale-space
Scale Space

Multiple smooth versions of a signal

Zero-crossings at multiple scale
Scale Space
Scale Space (Witkin, IJCAI 1983)

- Apply whole spectrum of scales
- Plot zero-crossings vs scales in a scale-space
- Interpret scale space contours
  - Contours are arches, open at the bottom, closed at the top
  - Interval tree
    - Each interval corresponds to a node in a tree, whose parent node represents larger interval, from which interval emerged, and whose off springs represent smaller intervals.
    - Stability of a node is a scale range over which the interval exits.
Scale Space

- Top level description
  - Iteratively remove nodes from the tree,
    - splicing out nodes that are less stable than any of their parents and off springs
A top level description of several signals using stability criterion.
Laplacian-of-Gaussian (LoG)

- Interest points:
  Local maxima in scale space of Laplacian-of-Gaussian

\[ L_{xx}(\sigma) + L_{yy}(\sigma) \rightarrow (x, y, \sigma) \]

\( \Rightarrow \) List of \((x, y, \sigma)\)

K. Grauman, B. Leibe
What Is A Useful Signature Function?

- Laplacian-of-Gaussian = “blob” detector
Scale-space blob detector: Example

Source: Lana Lazebnik
Scale-space blob detector: Example

source: Lana Lazebnik
Scale-space blob detector: Example
Building a Scale Space

- All scales must be examined to identify scale-invariant features.
- An efficient function is to compute the Laplacian Pyramid (Difference of Gaussian) (Burt & Adelson, 1983)
Approximation of LoG by Difference of Gaussians

\[ \frac{\partial G}{\partial \sigma} = \sigma \Delta^2 G \]

Heat Equation

\[ \sigma \Delta^2 G = \frac{\partial G}{\partial \sigma} = \frac{G(x, y, k\sigma) - G(x, y, \sigma)}{k\sigma - \sigma} \]

\[ G(x, y, k\sigma) - G(x, y, \sigma) \approx (k - 1)\sigma^2 \Delta^2 G \]

Typical values: \( \sigma = 1.6; \quad k = \sqrt{2} \)
Building a Scale Space

\[ G(x, y, k\sigma) = \frac{1}{2\pi(k\sigma)^2} e^{-(x^2+y^2)/(2k^2\sigma^2)} \]

\[ k = \sqrt{2} \]
Building a Scale Space

\[ G(x, y, k\sigma) = \frac{1}{2\pi (k\sigma)^2} e^{-\frac{(x^2 + y^2)}{2k^2\sigma^2}} \]

\[ k = \sqrt{2} \]
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<th>octave</th>
<th>scale</th>
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</tr>
<tr>
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</tr>
<tr>
<td>2.828427</td>
<td>4.000000</td>
</tr>
<tr>
<td>5.656854</td>
<td>8.000000</td>
</tr>
</tbody>
</table>

\[
\sigma = .707187.6; \quad k = \sqrt{2}
\]
How many scales per octave?
Initial value of sigma

![Graph showing repeatability (%) vs prior smoothing for each octave (sigma). The graph compares matching location and scale with the nearest descriptor in the database.](image-url)
Scale Space Peak Detection

- Compare a pixel ($X$) with 26 pixels in current and adjacent scales (Green Circles)
- Select a pixel ($X$) if larger/smaller than all 26 pixels
- Large number of extrema, computationally expensive
  - Detect the most stable subset with a coarse sampling of scales
Key Point Localization

- Candidates are chosen from extrema detection

original image

extrema locations
Initial Outlier Rejection

1. Low contrast candidates
2. Poorly localized candidates along an edge
   - Taylor series expansion of DOG, $D$.
   - Minima or maxima is located at $x = (x, y, \sigma)^T$
   - Value of $D(x)$ at minima/maxima must be large, $|D(x)| > \text{th}$.
Initial Outlier Rejection

from 832 key points to 729 key points, \( th=0.03 \).
Further Outlier Rejection

- DOG has strong response along edge
- Assume DOG as a surface
  - Compute principal curvatures (PC)
  - Along the edge one of the PC is very low, across the edge is high
Further Outlier Rejection

- Analogous to Harris corner detector
- Compute Hessian of $D$

\[
H = \begin{bmatrix}
D_{xx} & D_{xy} \\
D_{xy} & D_{yy}
\end{bmatrix}
\]

\[
Tr(H) = D_{xx} + D_{yy} = \lambda_1 + \lambda_2
\]

\[
Det(H) = D_{xx}D_{yy} - (D_{xy})^2 = \lambda_1 \lambda_2
\]

- Remove outliers by evaluating

\[
\frac{Tr(H)^2}{Det(H)} = \frac{(r+1)^2}{r}
\]

\[
r = \frac{\lambda_1}{\lambda_2}
\]

\[
\frac{Tr(H)^2}{Det(H)} = \frac{(\lambda_1 + \lambda_2)^2}{\lambda_1 \lambda_2} = \frac{(r\lambda_2 + \lambda_2)^2}{r\lambda_2} = \frac{(r+1)^2}{r}
\]
Further Outlier Rejection

- Following quantity is minimum (eigen values are equal) when $r=1$
- It increases with $r$

$$\frac{Tr(H)^2}{Det(H)} \leq \frac{(r+1)^2}{r}$$

- Eliminate key points if $r > 10$

$$\frac{Tr(H)^2}{Det(H)} < \frac{(r+1)^2}{r}$$
Further Outlier Rejection

from 729 key points to 536 key points.
Orientation Assignment

- To achieve rotation invariance
- Compute central derivatives, gradient magnitude and direction of $L$ (smooth image) at the scale of key point $(x,y)$

$$m(x, y) = \sqrt{(L(x + 1, y) - L(x - 1, y))^2 + (L(x, y + 1) - L(x, y - 1))^2}$$

$$\theta(x, y) = \tan^{-1}(((L(x, y + 1) - L(x, y - 1)))/(L(x + 1, y) - L(x - 1, y)))$$
Orientation Assignment

- Create a weighted direction histogram in a neighborhood of a key point (36 bins)

- Weights are
  - Gradient magnitudes
  - Spatial gaussian filter with $\sigma = 1.5 \times <\text{scale of key point}>$
Orientation Assignment

- Select the peak as direction of the key point
- Introduce additional key points (same location) at local peaks (if within 80% of max peak) of the histogram with different directions
Local Image Descriptors at Key Points

- Possible descriptor
  - Store intensity samples in the neighborhood
  - Sensitive to lighting changes, 3D object transformation

- Use of gradient orientation histograms
  - Robust representation
Similarity to IT cortex

- Complex neurons respond to a gradient at a particular orientation.
- Location of the feature can shift over a small receptive field.
• Edelman, Intrator, and Poggio (1997)
  • The function of the cells allow for matching and recognition of 3D objects from a range of viewpoints.
  • Experiments show better recognition accuracy for 3D objects rotated in depth by up to 20 degrees
Extraction of Local Image Descriptors at Key Points

- Compute relative orientation and magnitude in a 16x16 neighborhood at key point
- Form weighted histogram (8 bin) for 4x4 regions
  - Weight by magnitude and spatial Gaussian
  - Concatenate 16 histograms in one long vector of 128 dimensions
- Example for 8x8 to 2x2 descriptors
Descriptor Regions (n by n)
Extraction of Local Image Descriptors at Key Points

- Store numbers in a vector
- Normalize to unit vector ($\mathbf{UN}$)
  - Illumination invariance (affine changes)
- For non-linear intensity transforms
  - Bound Unit Vector items to maximum 0.2 (remove gradients larger than 0.2)
  - Renormalize to unit vector
Key point matching

- Match the key points against a database of that obtained from training images.
- Find the nearest neighbor i.e. a key point with minimum Euclidean distance.
  - Efficient Nearest Neighbor matching
    - Looks at ratio of distance between best and 2\textsuperscript{nd} best match (0.8)
Matching local features

Kristen Grauman
Matching local features

• To generate **candidate matches**, find patches that have the most similar appearance or SIFT descriptor

• Simplest approach: compare them all, take the closest (or closest k, or within a thresholded distance)
\[ V_{flag}(d_i) = \begin{cases} 
1; & \frac{|d_i - NN(d_i, 1)|}{|d_i - NN(d_i, 2)|} < 0.8 \\
0; & \text{otherwise} 
\end{cases} \]
Ambiguous matches

- At what distance do we have a good match?
- To add robustness to matching, can consider ratio: distance to best match / distance to second best match
  - If low, first match looks good.
  - If high, could be ambiguous match.

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The ratio of distance from the closest to the distance of the second closest
SIFT Detector

- Generate Scale Space of an Image
- Detect Peaks in Scale Space (extrema)
- Localize Interest Points (Taylor Series)
- Remove outliers (remove response along edges)
- Assign Orientation
SIFT Descriptor

- Compute relative orientation and magnitude in a 16x16 neighborhood at key point
- Form weighted histogram (8 bin) for 4x4 regions
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Reference